ANALYSIS OF MOTION OF A SURFACE THERMIC

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It is known that a sudden spherical discharge of gas of nonzero buoyancy rises and transforms into a vortex ring [1-10]. The transformation process has been studied theoretically and experimentally in [2-8], while numerical methods have been applied in [9, 10]. We will consider motion of axially symmetric thermics of hemispherical and cylindrical forms initially adjoining a horizontal surface.

We will consider a homogeneous medium with a density of ρ_0, bounded by the plane S, and in a gravitational field g perpendicular to S in which at t = 0 an axially symmetric free volume Q containing a medium with a density of $\rho_1 < \rho_0$ is generated. The boundary of Q is the surface F (at t = 0, a part of this surface coincides with the plane S). We will introduce a cylindrical coordinate system (z, r) whose z axis coincides with the axis of symmetry of the problem and is directed upward, while the origin of the coordinate system is situated at the point of intersection of the z-axis and the surface S.

We will consider motion of thermics whose boundaries at t = 0 are given by

$$z - h = 0,\ z = 0 \text{ for } r < R_0, \qquad 0 < z < h \text{ for } r = R_0 \tag{1}$$

(for a cylindrical thermic, where h is the height of the thermic and R_0 is the radius of its base)

$$z^2 + r^2 = R_0^2 \text{ for } z \geqslant 0 \tag{2}$$

(for a hemispherical thermic, where R_0 is the radius of its base).

At the points where the surfaces S and F intersect, there is a hydrostatic pressure gradient $\Delta p = \xi\rho_0 gH$, where $\xi = (\rho_0 - \rho_1)/\rho_0$ is the relative density gradient, H = h for a cylindrical thermic, and $H = R_0$ for a hemispherical thermionic. This pressure gradient is initially compensated by inertial forces, while for t > 0 boundary layer flow separating the thermic from the surface S is created. This situation is fully analogous to that considered in [3], where the difference in the hydrostatic pressure inside and outside the thermic forms a central stream out of the internal medium that penetrates the spherical thermic from below. The velocity of the top section of the flow v we will determine according to the Cauchy–Lagrange equations

$$\rho_0\varphi_t - \rho_1\varphi_{1t} + (1/2)\rho_0 v^2 = (\rho_0 - \rho_1)gH. \tag{3}$$

Here, φ and φ_1 are the potentials of the velocity inside and outside the thermic; the subscript t denotes differentiation with respect to time.

After initial lifting, the contribution of the inertial forces becomes relatively small, and the motion acquires a quasi-steady character. It is shown below that experiments confirm this assumption. Ignoring φ_t and φ_{1t} in Eq. (3), we have the following for the velocity of the top section of the flow along the surface S

$$v = \sqrt{2g\xi H}. \tag{4}$$

We obtain the following from (4) for the motion of the top section of the flow along the surface S

$$(R_0 - r)/H = t\sqrt{2g\xi/H} \quad \text{or} \quad R_0^0 - r^0 = t^0\sqrt{2}, \tag{5}$$

where the following coordinates are introduced:

$$r^0 = r/H,\ t^0 = t\sqrt{\xi g/H}. \tag{6}$$

Further evolution of the thermic has been studied experimentally. Experiments were conducted in a hermetically sealed basin 1 (Fig. 1) with the dimensions 1.2 × 1.2 × 5 m and with transparent lateral walls filled

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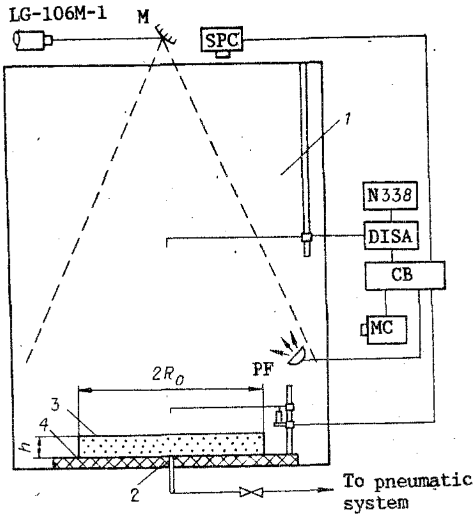

Fig. 1

with air. On the bottom of the basin, a system 2 has been positioned for obtaining surface thermics, which are created by breakdown of a soap-film bubble 3 (of cylindrical or hemispherical form) initially adjoining the horizontal surface 4 and filled with a nitrogen–helium mixture of a certain density. A small quantity of tobacco smoke is added to the volume of the thermic. The pattern of the flow is visually observed with a knife-shaped beam from the LG-106M-1 laser, which is fan-shaped due to the use of a convex cylindrical mirror (M), and the motion of the flow is captured on the film of the KONVAS-automatic movie camera (MC). In addition, a photographic flash PF has been placed in the lower part of the basin, while a still-picture camera SPC has been mounted on the transparent roof of the basin, which is pulse-controlled and is used for obtaining pictures of the motion of the thermic upward.

The circulation of the vortex ring formed by the thermic is determined with a thermoanemometer made by DISA, whose probe is situated above the thermic on the axis of symmetry. The thermoanemometer signal is registered by the N338 recorder. Triggering and operation of the whole system is done automatically according to a predetermined program. A control block (CB) is used for automation. A more detailed description of the system and the experimental methodology may be found in [8, 11].

In the experiments, the relative density gradient of the gas in the thermic was varied ($0.1 \le \xi \le 0.85$), as was its form and geometric characteristics (R_0 = 5-10 cm, h = 1-10 cm).

Movies made of the evolution of initially cylindrical and hemispherical thermics are given in Fig. 2 (in a and b, using a laser beam to illuminate the vertical cross section of the axially symmetric motion, while in c, examples of photographs from above (illuminated with the photographic flash). As is evident in Figs. 2a and 2b, a boundary layer flow of external medium is initially generated, which separates the thermic from the surface.

A plot of the motion of the top section of this flow is shown in Fig. 3 (a represents a cylindrical thermic in dimensionless coordinates (6), H = h; b is for a hemispherical thermic in the same dimensionless coordinates (6), H = R_0).

After initial lifting, the experimental points are grouped along the line representing the theoretical dependence (5). It is evident in Fig. 2 that as the top section of the boundary layer flow of external medium approximates the axis of symmetry, the flow separates from the surface S. Up to this moment, deceleration of the flow is observed (see Fig. 3, where the slope changes at the point A on the plot).

Simultaneously with the creation of flow of the external medium, which separates the thermic from the surface S, a vortex layer is generated on the surface F (as is also the case for an initially spherical thermic [3]), whose intensity is determined by the Bjerknes theorem [12]

$$\frac{d\Gamma}{dt} = \oint \frac{dp}{\rho}. \qquad (7)$$

After separation, the boundary layer flow rotates upward and penetrates the thermic. Hence, the latter rotates in the vortex ring. The parameters of the generated vortex differ from the corresponding parameters

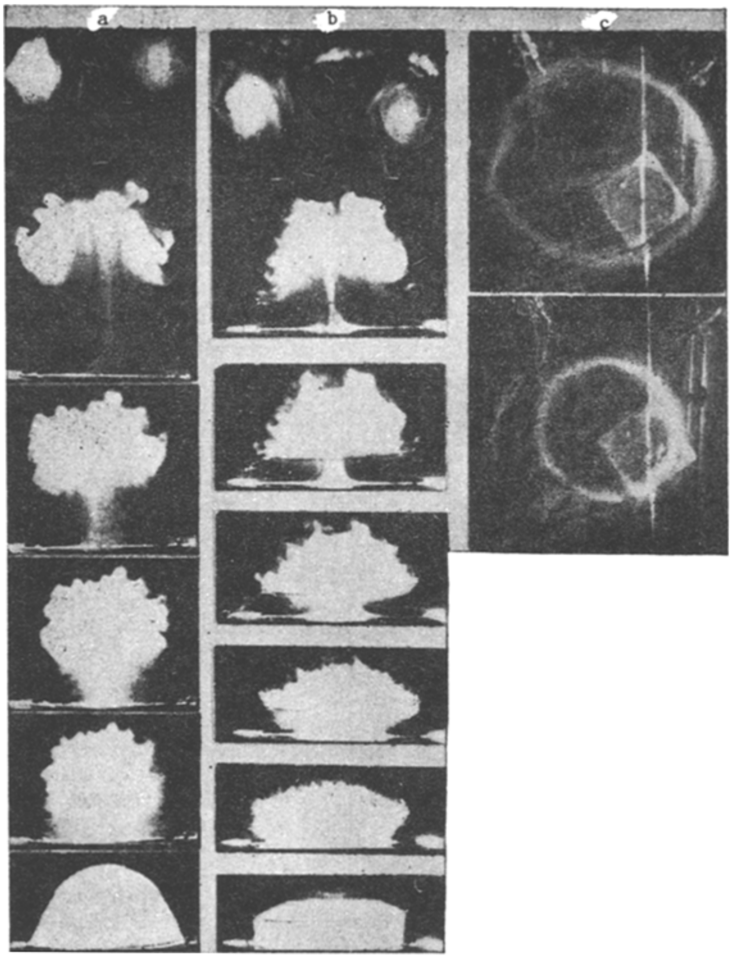

Fig. 2

of the vortex ring due to lift of the initially spherical thermic, but the general behavior of the motion in both cases, as is shown by experiments, is similar.

During the experiments, circulation of the generated vortex ring was measured by the same method used in [11]. The probe of the thermoanemometer was positioned at a height of ~1 m above the plane (i.e., at a height on the order of the largest characteristic dimension of the thermic). It was used for measuring the velocity at the center of the vortex ring. Movies and photographs from above make it possible to determine the geometric characteristics of the ring and its position relative to the thermoanemometer probe during motion. Circulation of the vortex ring is determined by the equation [11]

$$\Gamma = 2Ru(1 - 0.75\delta^2), \tag{8}$$

where R is the radius of the vortex ring; u is the value of the velocity determined by the thermoanemometer; $\delta = l/R$; and l is the distance between the center of the ring and the thermoanemometer probe.

The following quantity is illustrated in Fig. 4

$$\Gamma^0 = \frac{\Gamma}{\sqrt{\frac{3}{4\pi} V_0 g \xi}} \tag{9}$$

as a function of h/R_0 for a cylindrical thermic (a) and of ξ for an hemispherical thermic (b); V_0 is the initial volume of the thermic, which is equal to $\pi h R_0^2$ for a cylindrical thermic and $(2/3)\pi R_0^3$ for a hemispherical thermic. In the experiments, V_0 was varied over the range from 150 to 2500 cm^3.

We will estimate the quantity of the circulation Γ according to $\Gamma = \oint v dl = v_{av} L$, where L is the length of the contour bounding the vortex tube and v_{av} is the velocity of the flow at one of the points on this contour (the characteristic velocity).

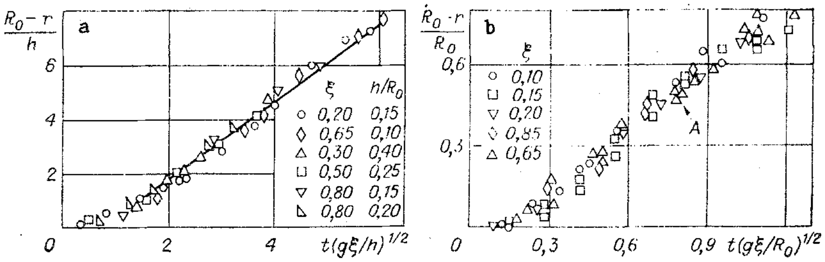

Fig. 3

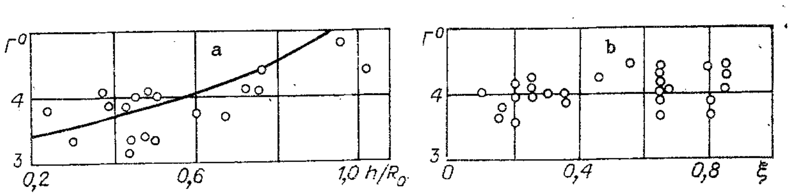

Fig. 4

We will assume $v_{av} \simeq v = (2g\xi h)^{1/2}$ for a cylindrical thermic, while the length of the contour is the perimeter of the ellipse formed between the z axis and the boundary of the thermic, i.e., [13]

$$L = (\pi R_0/2)(1 + \beta)(1 + \lambda^2/4 + \lambda^4/64 + \ldots),$$
$$\beta = h/R_0,\ \lambda = (1 - \beta)/(1 + \beta).$$

Then

$$\Gamma^0 = \pi\sqrt{\frac{2}{3}}(1+\beta)\left(1+\frac{\lambda^2}{4}+\frac{\lambda^4}{64}+\ldots\right). \tag{10}$$

For a hemispherical thermic, we have (see Fig. 3b) $v_{av} \simeq 0.62(2g\xi R_0)^{1/2}$, and the length of the contour is the length of the circle inscribed in half the semicircle: $L = 2\pi R_0/(1+\sqrt{2})$.

We obtain the following estimate for Γ^0:

$$\Gamma^0 = 5\pi/[2(1+\sqrt{2})] \sim 4. \tag{11}$$

Equations (10) and (11) are shown in Fig. 4 by the curves, and the experimental points lie somewhat lower, which is due to the fact that $v_{av} < p$.

Experimental plots are given in Fig. 5 for the motion of the vortex rings created by initially cylindrical surface thermics, and curve 1 from [11] is indicated, which corresponds to motion of vortex rings formed by initially spherical thermics. The experimental points are grouped lower, which is due to the small value of Γ^0 for the vortex ring formed by a cylindrical thermic [see Fig. 4a and Eq. (10)] in comparison to the value of Γ^0 for a vortex ring formed by an initially spherical thermic [11].

Experiments indicate that a volume of gas of nonzero buoyancy initially adjoining a horizontal surface rises, separates from the surface, concentrates into a compact mass, and transforms into a vortex ring. The evolution process of the motion may be divided into three steps: creation of a surface radial flow of external medium, which separates the thermic from the surface, formation of a rising vortex ring, and lift of the vortex ring. In the experiments, the vortex ring, which lifted to a height of $\sim 40R_*$ [$R_* = (3V_0/4\pi)^{1/3}$], lost its stability and broke down, which may have been due to the effect of the lateral walls of the experimental basin.

Surface flow is generated and maintained for a certain length of time due to the difference in hydrostatic pressures at the level of the surface S inside and outside the thermic, which is confirmed by a plot of the motion of the top section in Fig. 3, where it is evident that the experimental data agree well with those calculated with Eq. (5).

When the top section approximates the axis of symmetry, the flow is decelerated and separates from the surface. During separation and rotation of the surface flow near the point of intersection of the axis of sym-

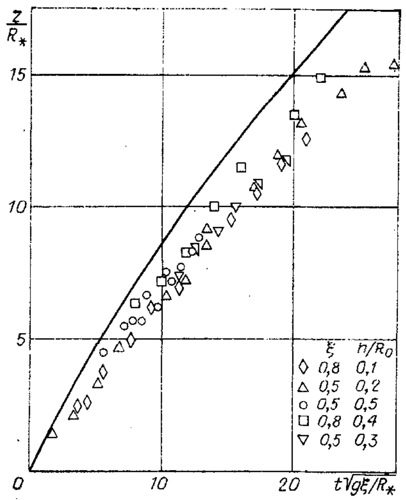

Fig. 5

metry and the plane S, the central section of the thermic forms a motionless zone (of reduced pressure), which is preserved during the lift of the thermic up to the point where the dynamic pressure gradient $p - p_{01} = \frac{1}{2}\rho_0 v^2$ (p_{01} is the pressure at the surface S near the boundary zone) is no less than the static pressure $\xi\rho_0 gz$. The remaining gas rises, transforming into a vortex ring. The transformation process also occurs for lifting of an initially spherical thermic [3, 8].

The free surface separating the motionless zone from the flow zone is unstable for equal densities, but if the motionless zone is filled with a light gas ($\xi > 0$), the position of this surface (the contact surface) becomes stable [12].

The motionless zone (this may be seen in Fig. 2 in the column-shaped volume filled with a light gas connecting the surface S with the thermic during lift) is preserved up to the height $H = (2\text{-}3)R_*$ and is a zone formed by the separation of the surface flow induced by the rising thermic.

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